A Concept Category Construction

Dusko Pavlovic
University of Hawaii

ongoing joint work with
Dominic Hughes
Apple Inc.

Logic Matters
December 28, 2021

Outline

Background: Mining concepts from data

Problem: From recommenders to echo chambers

Approach: Data dependencies as morphisms

Method: Nuclear adjunctions

Solution: The concept category

Architecture: Channels = adjunctions

Outline

Background: Mining concepts from data

Formal Concept Analysis

Latent Semantic Analysis

Problem: From recommenders to echo chambers

Approach: Data dependencies as morphisms

Method: Nuclear adjunctions

Solution: The concept category

Architecture: Channels = adjunctions

Concept mining

	NPR	CNN	FOX	KHON
Alice	★★★★	★★	★★★★	★
Bob	★★	★★★	★★	
Carol	★★	★	★★★★★	
Dave	★	★★★★	★★★	
Ed		★★	★★★★★	★★

A Recommender System mines concepts from data

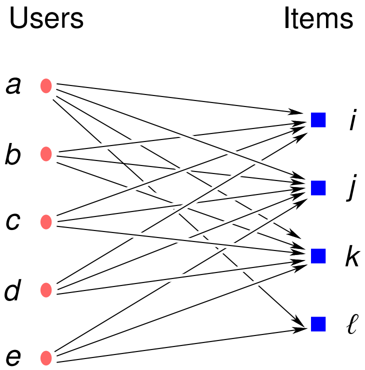
Concept mining is the goal of data analysis

domain	$\mathcal{J}$	$\mathcal{U}$	R	A_{iu}
user preference	items	users	$\{0, \ldots, 5\}$	rating
text analysis	documents	terms	$\mathbb{N}$	occurrence
topic search	authorities	hubs	$\mathbb{N}$	hyperlinks
measurement	instances	quantities	$\mathbb{R}$	outcome
concept analysys	objects	attributes	$\{0, 1\}$	property
elections	candidates	voters	$\{1, \ldots, n\}$	preference
market	producers	consumers	$\mathbb{Z}$	deliveries
digital images	positions	pixels	$[0, 1]$	intensity

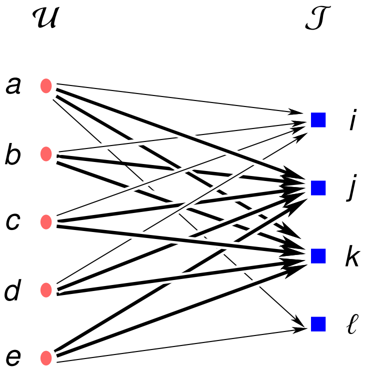
Formal Concept Analysis

	NPR	CNN	FOX	KHON
Alice	⋆	⋆	⋆	⋆
Bob	⋆	⋆	⋆	
Carol	⋆	⋆	⋆	
Dave	⋆	⋆	⋆	
Ed		⋆	⋆	⋆

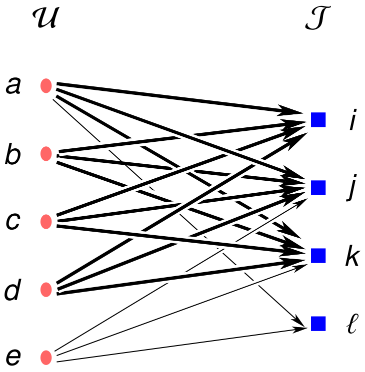
Formal Concept Analysis



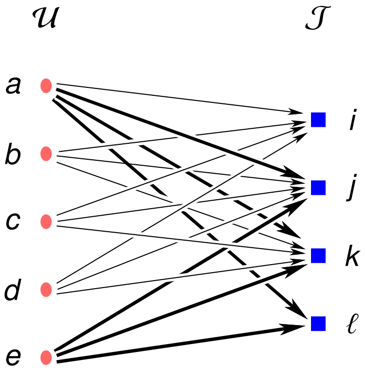
Concepts are complete subgraphs



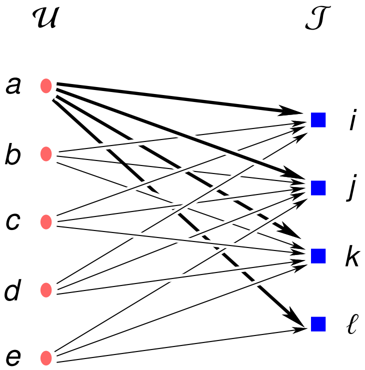
Concepts are complete subgraphs



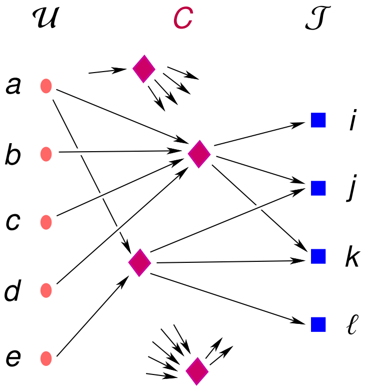
Concepts are complete subgraphs



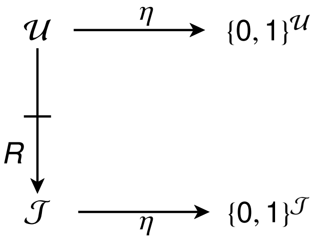
Concepts are complete subgraphs



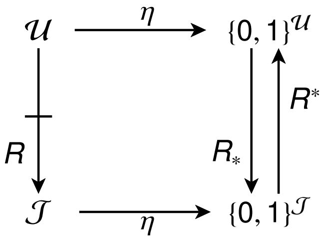
Concept lattice



Powersets are $\bigcup$-completions



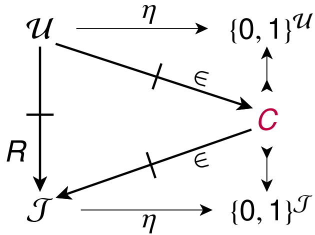
$\bigcup$- completions $\rightsquigarrow$ Galois connections



$$R_* X = \bigcap_{x \in X} xR \qquad \text{where } xR = \{y \in \mathcal{J} \mid xRy\}$$

$$R^* Y = \bigcap_{y \in Y} Ry \qquad \text{where } Ry = \{x \in \mathcal{U} \mid xRy\}$$

Fixpoints ⇝ tight bicompletion ⇝ concept lattice

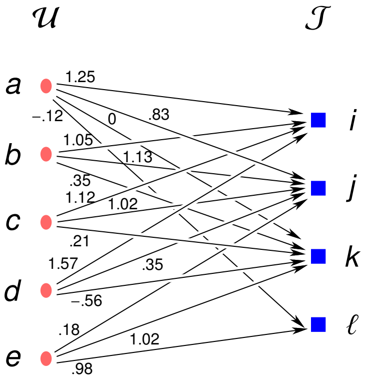


$$xRy \quad = \quad \bigvee_{c \in C} x \in c_{\mathcal{U}} \wedge c_{\mathcal{J}} \ni y$$

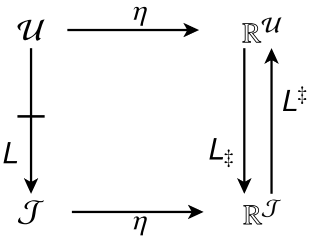
Latent Semantic Analysis

	a	b	c	d	e
i	1.25	1.05	1.12	1.57	
j	.83	1.13	1.02	.35	.18
k	0	.35	.21	-.56	1.02
ℓ	-.12				.98

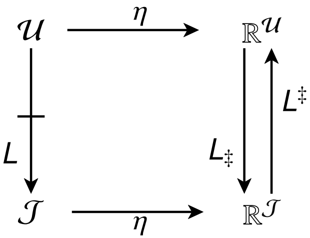
Latent Semantic Analysis



Vector spaces are mix-completions



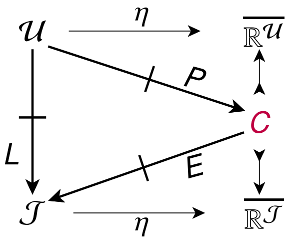
Matrices $\rightsquigarrow$ adjoint operators



$$L_{\ddagger}\xi = \left(\sum_{j=1}^{n} L_{ij}\xi_j\right)_{i=1}^{m}$$

$$L^{\ddagger}\upsilon = \left(\sum_{i=1}^{m} \upsilon_i L_{ij}\right)_{j=1}^{n}$$

Eigenspaces ⇝ tight completion ⇝ concept space



$$L_{ij} = \sum_{\gamma \in C} \lambda_\gamma \cdot E_{i\gamma} \cdot P_{\gamma j}$$

Spectrum $\rightsquigarrow$ dominant concepts

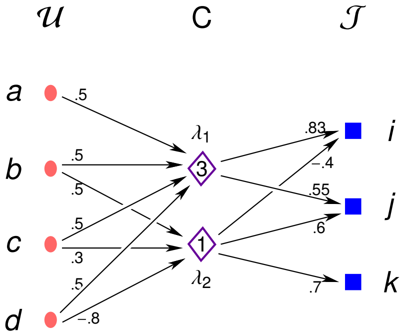
Concepts are the eigenspaces of $L_\ddagger L^\ddagger$ and $L^\ddagger L_\ddagger$, because

$$\gamma_{\mathcal{U}} = L^\ddagger \gamma_{\mathcal{J}} \quad \wedge \quad L_\ddagger \gamma_{\mathcal{U}} = \gamma_{\mathcal{J}}$$
$$\Updownarrow$$
$$\gamma_{\mathcal{U}} = L^\ddagger L_\ddagger \gamma_{\mathcal{U}} \quad \wedge \quad L_\ddagger L^\ddagger \gamma_{\mathcal{J}} = \gamma_{\mathcal{J}}$$

Singular Value Decomposition (SVD)

$$\begin{pmatrix} 1.25 & 1.05 & 1.12 & 1.57 \\ .83 & 1.13 & 1.02 & .35 \\ 0 & .35 & .21 & -.56 \end{pmatrix} = \begin{pmatrix} .83 & -.4 \\ .55 & .6 \\ 0 & .7 \end{pmatrix} \cdot \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} .5 & .5 & .5 & .5 \\ 0 & .5 & .3 & -.8 \end{pmatrix}$$

Information flows through concepts



Outline

Background: Mining concepts from data

Problem: From recommenders to echo chambers

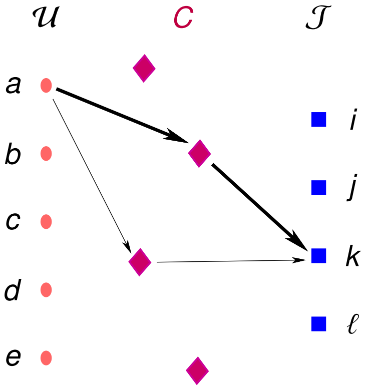
Approach: Data dependencies as morphisms

Method: Nuclear adjunctions

Solution: The concept category

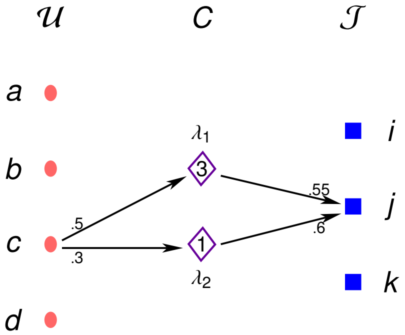
Architecture: Channels = adjunctions

FCA: Concepts are the **particles** of meaning



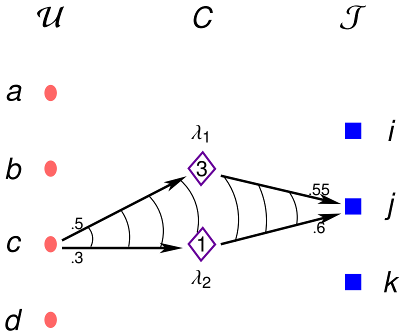
$$aRk \quad = \quad \exists c \in C.\ \ a \in c \ \wedge \ c \ni k$$

LSA: Concept associations add up. . .



$$L_{cj} = 3(.5 \times .55) + (.3 \times .6)$$

. . . and create **waves** of meaning



$$L_{cj} = 3(.5 \times .55) + (.3 \times .6)$$

Wave mechanics is the theory of interference.

The meaning of numbers

- L_{ui} = how much does the user u use the item i
- $P_{u\gamma}$ = how much of u's usage is due to the concept γ
- $E_{\gamma i}$ = how much of the utility of i is due to the concept γ

The meaning of decomposition

If the data are normalized

$$L_{ui} = \Pr(u, i) \qquad E_{\gamma i} = \Pr(\gamma, i) \qquad P_{u\gamma} = \Pr(u, \gamma)$$

then the decomposition $L_{ui} = \sum_\gamma \left(P_{u\gamma} \times E_{\gamma i}\right)$ implies

$$\begin{aligned} \Pr(u, i) &= \Pr(u, \gamma_1, i) + \Pr(u, \gamma_2, i) \\ &= \Pr(u, \gamma_1) \times \Pr(\gamma_1, i) + \Pr(u, \gamma_2) \times \Pr(\gamma_2, i) \end{aligned}$$

The meaning of decomposition

If the data are normalized

$$L_{ui} = \Pr(u,i) \qquad E_{\gamma i} = \Pr(\gamma,i) \qquad P_{u\gamma} = \Pr(u,\gamma)$$

then the decomposition $L_{ui} = \sum_\gamma \left(P_{u\gamma} \times E_{\gamma i}\right)$ implies

$$\begin{aligned} \Pr(u,i) &= \Pr(u,\gamma_1,i) + \Pr(u,\gamma_2,i) \\ &= \Pr(u,\gamma_1) \times \Pr(\gamma_1,i) + \Pr(u,\gamma_2) \times \Pr(\gamma_2,i) \end{aligned}$$

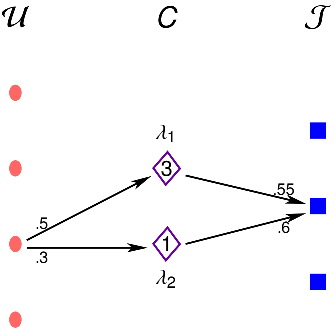
— which means that *u* and *i* are independent —

Recommendations invalidate their own independency assumption.

Any existing dependencies are amplified.

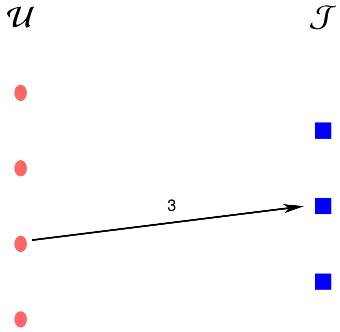
Outline

Concept associations add up



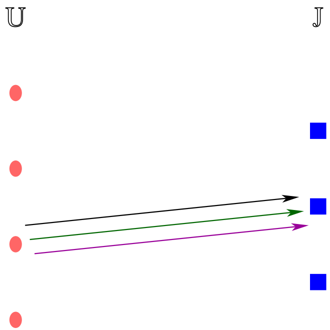
$$L_{ui} = \lambda_1 P_{u\gamma_1} E_{\gamma_1 i} + \lambda_2 P_{u\gamma_2} E_{\gamma_2 i}$$

... to explain the correlation counts



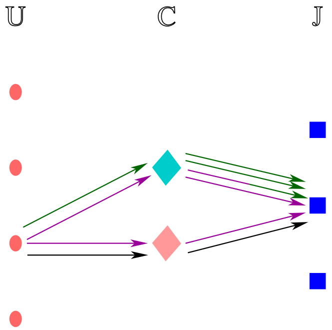
$$\begin{array}{rcl} L\colon \mathcal{U}\times\mathcal{I} & \longrightarrow & \mathbb{R} \\ \langle u,i\rangle & \mapsto & 3 \end{array}$$

Idea: Record correlation events



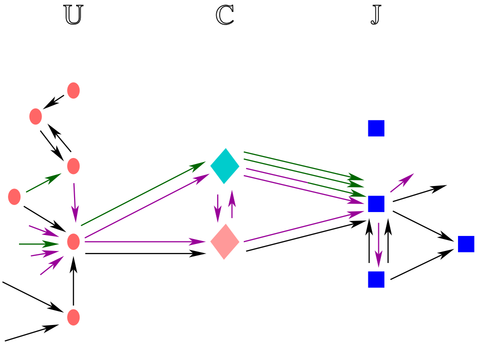
$$\begin{array}{rcl}\Phi : \mathbb{U}^o \times \mathbb{J} & \longrightarrow & \mathsf{Set} \\ \langle u, i \rangle & \mapsto & \{\uparrow, {\color{magenta}\uparrow}, {\color{green}\uparrow}\}\end{array}$$

Decomposition of set-matrices



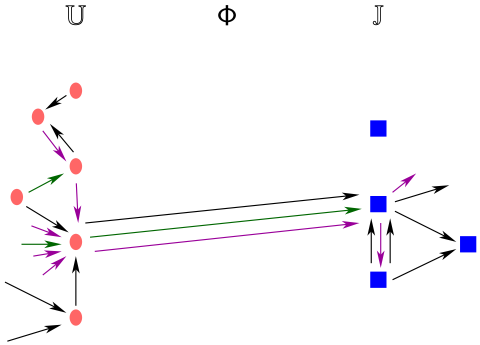
$$
\begin{array}{ccll}
\Phi(u,i) & \longleftarrow & \Upsilon(u,\gamma_1)\times\Psi(\gamma_1,i) & +\ \Upsilon(u,\gamma_2)\times\Psi(\gamma_2,i) \\
\uparrow & \leftarrow\!\shortmid & & \langle\uparrow,\uparrow\rangle \\
\uparrow & \leftarrow\!\shortmid & \langle\uparrow,\uparrow\uparrow\uparrow\rangle & \\
\uparrow & \leftarrow\!\shortmid & \langle\uparrow,\uparrow\uparrow\rangle, & \langle\uparrow,\uparrow\rangle
\end{array}
$$

Decomposition of *categorical* matrices



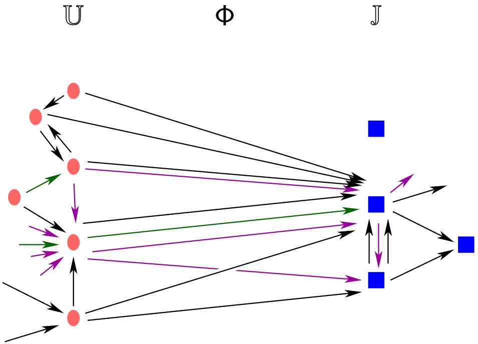
Known data correlations are captured in terms of morphisms

Categorical matrices
(a.k.a. distributors, profunctors, bimodules)



Events can be identified by identifying the morphism actions

Categorical matrices
(a.k.a. distributors, profunctors, bimodules)



Data dependencies can be captured in terms of morphism compositions.

Categorical matrix

is a matrix of sets acted upon by categories:

$$\mathbb{U}(a, a') \times \Phi(a', k') \times \mathbb{J}(k', k) \xrightarrow{\Phi} \Phi(a, k)$$

Categorical matrix composition

$$\mathbb{U}(u,u') \times \Upsilon(u',\gamma') \times \mathbb{C}(\gamma',\gamma) \xrightarrow{\Upsilon} \Upsilon(u,\gamma)$$

$$\mathbb{C}(\gamma,\gamma'') \times \Psi(\gamma'',i'') \times \mathbb{J}(i'',i) \xrightarrow{\Psi} \Psi(\gamma,i)$$

Categorical matrix composition

$$\frac{\mathbb{U}(u,u') \times \Upsilon(u',\gamma') \times \mathbb{C}(\gamma',\gamma) \xrightarrow{\Upsilon} \Upsilon(u,\gamma) \qquad \mathbb{C}(\gamma,\gamma'') \times \Psi(\gamma'',i'') \times \mathbb{J}(i'',i) \xrightarrow{\Psi} \Psi(\gamma,i)}{\mathbb{U}(u,u') \times \Upsilon(u',\gamma) \times \Psi(\gamma,i') \times \mathbb{J}(i',i) \xrightarrow{\Upsilon;\Psi} \Upsilon(u,\gamma) \times \Psi(\gamma,i)}$$

Categorical matrix composition

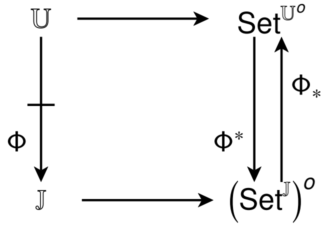
$$\dfrac{\dfrac{\mathbb{U}(u,u')\times\Upsilon(u',\gamma')\times\mathbb{C}(\gamma',\gamma)\xrightarrow{\Upsilon}\Upsilon(u,\gamma)\qquad\mathbb{C}(\gamma,\gamma'')\times\Psi(\gamma'',i'')\times\mathbb{J}(i'',i)\xrightarrow{\Psi}\Psi(\gamma,i)}{\mathbb{U}(u,u')\times\Upsilon(u',\gamma)\times\Psi(\gamma,i')\times\mathbb{J}(i',i)\xrightarrow{\Upsilon;\Psi}\Upsilon(u,\gamma)\times\Psi(\gamma,i)}}{\mathbb{U}(u,u')\times\Phi(u',i')\times\mathbb{J}(i',i)\xrightarrow{\Phi}\Phi(u,i)}$$

where

$$\begin{aligned}\Phi(u,i) &= \int_{\gamma\in\mathbb{C}}\Upsilon(u,\gamma)\times E(\gamma,i)\\ &= \left(\coprod_{\gamma\in\mathbb{C}}\Upsilon(u,\gamma)\times E(\gamma,i)\right)\Big/\sim\end{aligned}$$

for $\langle v,\psi\rangle\sim\exists u\exists j.\ \langle u^{*}v, j_{*}\psi\rangle$

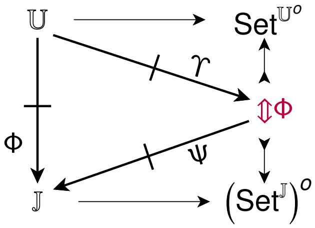
Categorical matrices $\rightsquigarrow$ adjoint functors



$$\Phi^* X = \varinjlim \Phi_u X_u$$

$$\Phi_* Y = \varprojlim Y_i \Phi_i$$

Task: tight completion ⇝ concept category



$$\Phi(u,i) \quad = \quad \int_{\gamma \in C} \Upsilon(u,\gamma) \times \Psi(\gamma,i)$$

Retrace the FCA workflow?

Posetal matrix

is a lower-closed set $R \subseteq \mathcal{U} \times \mathcal{I}^o$, i.e.

$$u \overset{\mathcal{U}}{\leq} u' \wedge u' R i' \wedge i' \overset{\mathcal{I}}{\leq} i \quad \Longrightarrow \quad uRi$$

Posetal matrix composition

$$\frac{\dfrac{u \overset{\mathcal{U}}{\leq} u' \wedge u'P\gamma' \wedge \gamma' \overset{C}{\leq} \gamma \implies uP\gamma \qquad \gamma \overset{C}{\leq} \gamma'' \wedge \gamma''Ei'' \wedge i'' \overset{\mathcal{I}}{\leq} i \implies \gamma Ei}{u \overset{\mathcal{U}}{\leq} u' \wedge u'P\gamma \wedge \gamma Ei' \wedge i' \overset{\mathcal{I}}{\leq} i \implies uP\gamma \wedge \gamma Ei}}{u \overset{\mathcal{U}}{\leq} u' \wedge u'Ri' \wedge i' \overset{\mathcal{I}}{\leq} i \implies uRi}$$

where

$$uRi \iff \exists \gamma.\, uP\gamma \wedge \gamma Ei$$

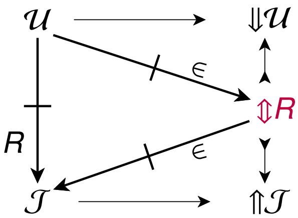
Posetal matrix composition

$$\frac{\dfrac{u \overset{\mathcal{U}}{\leq} u' \wedge u' \in \gamma' \wedge \gamma' \subseteq \gamma \implies u \in \gamma \qquad \gamma \subseteq \gamma'' \wedge \gamma'' \ni i'' \wedge i'' \overset{\mathcal{I}}{\leq} i \implies \gamma \ni i}{u \overset{\mathcal{U}}{\leq} u' \wedge u' \in \gamma \wedge \gamma \ni i' \wedge i' \overset{\mathcal{I}}{\leq} i \implies u \in \gamma \wedge \gamma \ni i}}{u \overset{\mathcal{U}}{\leq} u' \wedge u' R i' \wedge i' \overset{\mathcal{I}}{\leq} i \implies u R i}$$

where

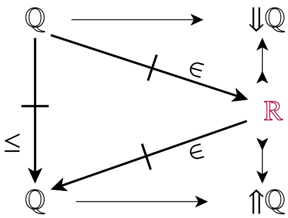
$$uRi \iff \exists \gamma.\ u \in \gamma \wedge \gamma \ni i$$

Tight completion $\rightsquigarrow$ concept lattice



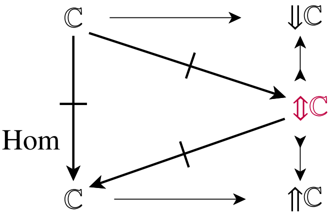
$$uRi \quad = \quad \exists\gamma \in C.\ u \in \gamma_{\Downarrow} \wedge \gamma_{\Uparrow} \ni i$$

Dedekind completion $\rightsquigarrow$ the real continuum

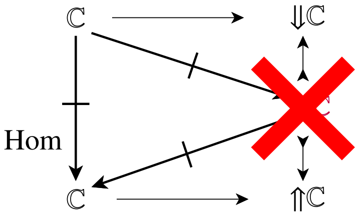


$$q \overset{\mathbb{Q}}{\leq} q' \quad = \quad \exists r \in \mathbb{R}.\ q \in r_{\Downarrow} \wedge r_{\Uparrow} \ni q'$$

Dedekind completion of a category
(Lambek 1964)



Dedekind completion of a category doesn't exist
(Isbell 1972)



But we use and compute concept categories

Upshot

- **Background:** Tight poset completions for $\bigvee \leftrightsquigarrow \bigwedge$.
- **Task:** Tight category completions.
- **Obstacle:** No tight category completions for $\varprojlim \not\leftrightsquigarrow \varinjlim$.
- **New task:** Tight category completions for
  tight limits $\overleftarrow{\lim} \leftrightsquigarrow \overrightarrow{\lim}$.

Upshot

- **Background:** Tight poset completions for $\bigvee \leftrightsquigarrow \bigwedge$.
- **Task:** Tight category completions.
- **Obstacle:** No tight category completions for $\varprojlim \not\leftrightsquigarrow \varinjlim$.
- **New task:** Tight category completions for
  tight limits $\overleftarrow{\lim} \leftrightsquigarrow \overrightarrow{\lim}$.
  - What are $\overleftarrow{\lim} \leftrightsquigarrow \overrightarrow{\lim}$?

Outline

Reminder: Meets and joins

$$\mathbb{P} \underset{\triangledown}{\overset{\bigvee}{\leftrightarrows}} \Downarrow\mathbb{P} \quad (\bot)$$

$$\mathbb{P}(\varinjlim \overleftarrow{\alpha}, y) \cong \Downarrow\mathbb{P}(\overleftarrow{\alpha}, \triangledown y)$$

$$\mathbb{P} \underset{\vartriangle}{\overset{\bigwedge}{\leftrightarrows}} \Uparrow\mathbb{P} \quad (\top)$$

$$\mathbb{P}(x, \varprojlim \overrightarrow{\beta}) \cong \Uparrow\mathbb{P}(\vartriangle x, \overrightarrow{\beta})$$

Reminder: Limits and colimits

$$\mathbb{C} \underset{\nabla}{\overset{\underrightarrow{\lim}}{\underset{\perp}{\leftrightarrows}}} \Downarrow\mathbb{C}$$

$$\mathbb{C}(\underrightarrow{\lim}\,\overleftarrow{\alpha}, y) \cong \Downarrow\mathbb{C}(\overleftarrow{\alpha}, \nabla y)$$

$$\mathbb{C} \underset{\vartriangle}{\overset{\underleftarrow{\lim}}{\underset{\top}{\leftrightarrows}}} \Uparrow\mathbb{C}$$

$$\mathbb{C}(x, \underleftarrow{\lim}\,\overrightarrow{\beta}) \cong \Uparrow\mathbb{C}(\vartriangle x, \overrightarrow{\beta})$$

Tight limits and colimits

$$\mathbb{C} \underset{\nabla}{\overset{\overrightarrow{\lim}}{\underset{\bot}{\leftrightarrows}}} \Downarrow\mathbb{C}^{\circlearrowleft} \qquad\qquad \mathbb{C} \underset{\Delta}{\overset{\overleftarrow{\lim}}{\underset{\top}{\leftrightarrows}}} \Uparrow\mathbb{C}^{\circlearrowleft}$$

$$\mathbb{C}(\underset{\varphi}{\overrightarrow{\lim}}\, \overleftarrow{\alpha}, y) \cong \Downarrow\mathbb{C}^{\circlearrowleft}(\varphi_{\overleftarrow{\alpha}}, \mathrm{id}_{\nabla y}) \qquad \mathbb{C}(x, \underset{\psi}{\overleftarrow{\lim}}\, \overrightarrow{\beta}) \cong \Uparrow\mathbb{C}^{\circlearrowleft}(\mathrm{id}_{\Delta x}, \psi_{\overrightarrow{\beta}})$$

where. . .

Tight limits and colimits

where $\Downarrow\mathbb{C}^{\circlearrowleft}$ is defined

$$|\Downarrow\mathbb{C}^{\circlearrowleft}| = \coprod_{\overleftarrow{\alpha}\in\Downarrow\mathbb{C}} \left\{ \begin{array}{ccc} \underline{\Delta}\overleftarrow{\alpha} & \overline{\nabla}\underline{\Delta}\overleftarrow{\alpha} \xrightarrow{\qquad\qquad} & \alpha \\ \varphi\downarrow & \searrow^{\varphi} \qquad \searrow^{\overline{\nabla}\varphi} & \downarrow \\ \underline{\Delta}\overleftarrow{\alpha} \xrightarrow{\varphi} & \underline{\Delta}\overleftarrow{\alpha} & \overline{\nabla}\underline{\Delta}\overleftarrow{\alpha} \end{array} \right\}$$

$$\Downarrow\mathbb{C}^{\circlearrowleft}(\varphi_{\overleftarrow{\alpha}}, \psi_{\overleftarrow{\beta}}) = \left\{ f \in \Downarrow\mathbb{C}(\overleftarrow{\alpha}, \overleftarrow{\beta}) \;\middle|\; \begin{array}{ccc} \underline{\Delta}\overleftarrow{\alpha} & \xrightarrow{\underline{\Delta}f} & \underline{\Delta}\overleftarrow{\beta} \\ \varphi\downarrow & & \downarrow\psi \\ \underline{\Delta}\overleftarrow{\alpha} & \xrightarrow{\underline{\Delta}f} & \underline{\Delta}\overleftarrow{\beta} \end{array} \right\}$$

and $\Uparrow\mathbb{C}^{\circlearrowleft}$ is dual.

(Background proposition)

$$\Downarrow\mathbb{C}^{\circlearrowleft} \simeq \Uparrow\mathbb{C}^{\boxbslash} \qquad\qquad \Uparrow\mathbb{C}^{\circlearrowleft} \simeq \Downarrow\mathbb{C}^{\boxslash}$$

follows from

Quotients in monadic programming: Projective algebras are equivalent to coalgebras. LICS 2017 or `https://arxiv.org/abs/1701.07601`

Outline

Background: Mining concepts from data

Problem: From recommenders to echo chambers

Approach: Data dependencies as morphisms

Method: Nuclear adjunctions

Solution: The concept category

Architecture: Channels = adjunctions

Concept category

$$
|\Updownarrow\Phi| \;=\; \coprod_{\substack{\overleftarrow{\alpha}\in\Downarrow\mathbb{U}^{\overleftarrow{\Phi}}\\ \overrightarrow{\alpha}\in\Uparrow\mathbb{J}^{\overrightarrow{\Phi}}}} \left\{ \overleftarrow{\alpha} \underset{\overleftarrow{\jmath}}{\overset{\overleftarrow{g}}{\twoheadleftarrow\!\cdots\!\rightarrowtail}} \Phi_{\sharp}\overrightarrow{\alpha} \quad\wedge\quad \Phi^{\sharp}\overleftarrow{\alpha} \underset{\overleftarrow{\jmath}}{\overset{\overrightarrow{g}}{\twoheadrightarrow\!\cdots\!\leftarrowtail}} \overrightarrow{\alpha} \right\}
$$

$$
\Updownarrow\Phi(\alpha,\beta) \;=\; \coprod_{\substack{\overleftarrow{f}\in\Downarrow\mathbb{U}^{\overleftarrow{\Phi}}(\overleftarrow{\alpha},\overleftarrow{\beta})\\ \overrightarrow{f}\in\Uparrow\mathbb{J}^{\overrightarrow{\Phi}}(\overrightarrow{\alpha},\overrightarrow{\beta})}} \left\{ \begin{array}{ccccc} \overleftarrow{x} & \rightarrowtail\overleftarrow{\jmath}_{\alpha}\rightarrow & \Phi_{*}\overrightarrow{x} & -\overleftarrow{g}_{\alpha}\twoheadrightarrow & \overleftarrow{x} \\ \Big|\,\overleftarrow{f} & & \Big|\,\Phi_{*}\overrightarrow{f} & & \Big|\,\overleftarrow{f} \\ \downarrow & & \downarrow & & \downarrow \\ \overleftarrow{y} & \rightarrowtail\overleftarrow{\jmath}_{\beta}\rightarrow & \Phi_{*}\overrightarrow{y} & -\overleftarrow{g}_{\beta}\twoheadrightarrow & \overleftarrow{y} \end{array} \right\}
$$

Concept category

$$|\Updownarrow\Phi| \;=\; \coprod_{\substack{\overleftarrow{\alpha}\in\Downarrow\mathbb{U}^{\overleftarrow{\Phi}}\\ \overrightarrow{\alpha}\in\Uparrow\mathbb{J}^{\overrightarrow{\Phi}}}} \left\{ \overleftarrow{\alpha} \underset{\overleftarrow{\jmath}}{\overset{\overleftarrow{g}}{\twoheadleftarrow\!\!\cdots\!\!\rightarrowtail}} \Phi_\sharp\overrightarrow{\alpha} \quad\wedge\quad \Phi^\sharp\overleftarrow{\alpha} \underset{\overrightarrow{\jmath}}{\overset{\overrightarrow{g}}{\leftarrowtail\!\!\cdots\!\!\twoheadrightarrow}} \overrightarrow{\alpha} \right\}$$

$$\Updownarrow\Phi(\alpha,\beta) \;=\; \coprod_{\substack{\overleftarrow{f}\in\Downarrow\mathbb{U}^{\overleftarrow{\Phi}}(\overleftarrow{\alpha},\overleftarrow{\beta})\\ \overrightarrow{f}\in\Uparrow\mathbb{J}^{\overrightarrow{\Phi}}(\overrightarrow{\alpha},\overrightarrow{\beta})}} \left\{ \begin{array}{ccccc} \overrightarrow{x} & \twoheadleftarrow \overrightarrow{g}_\alpha - & \Phi^*\overleftarrow{x} & \leftarrow \overrightarrow{\jmath}_\alpha \prec & \overrightarrow{x} \\ \uparrow & & \uparrow & & \uparrow \\ \overrightarrow{f} & & \Phi^*\overleftarrow{f} & & \overrightarrow{f} \\ | & & | & & | \\ \overrightarrow{y} & \twoheadleftarrow \overrightarrow{g}_\beta - & \Phi^*\overleftarrow{y} & \leftarrow \overrightarrow{\jmath}_\beta \prec & \overrightarrow{y} \end{array} \right\}$$

Background

- Toshiki Kataoka and DP, *Towards concept analysis in categories*. CALCO 2015 or https://arxiv.org/abs/2004.07353
- DP and P.M. Seidel, *Quotients in monadic programming: Projective algebras are equivalent to coalgebras*. LICS 2017 or https://arxiv.org/abs/1701.07601
- DP and D. Hughes, *The nucleus of an adjunction and the Street monad on monads*. https://arxiv.org/abs/2004.07353

Outline

Background: Mining concepts from data

Problem: From recommenders to echo chambers

Approach: Data dependencies as morphisms

Method: Nuclear adjunctions

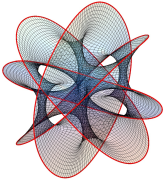
Solution: The concept category

Architecture: Channels = adjunctions

Semantics

$\models$ COW

Semantics in mathematics = category theory



$$\models \; Hom(\Delta, \mathcal{M})$$

Semantics is adjunction

$$Mnf(\mathcal{M}, R\Delta) \;\cong\; Grp(L\mathcal{M}, \Delta)$$

Sign learners

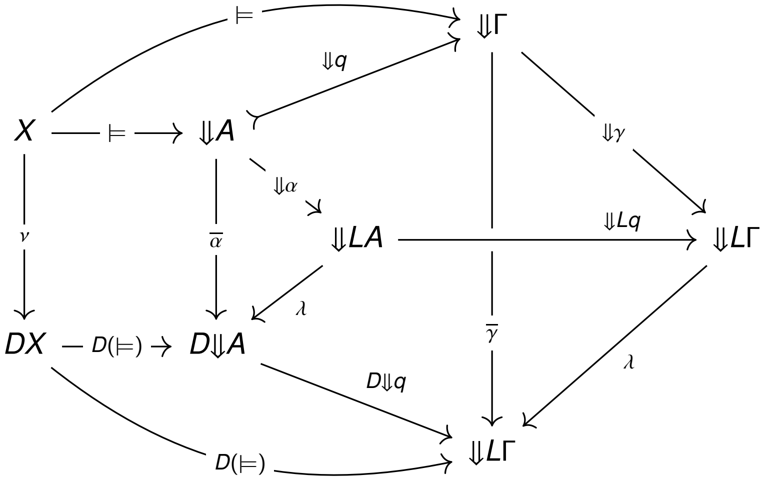
- **children:** competent learning
- **cryptanalysts:** adversarial learning
- **scientists:** approximate learning

Sign is a process

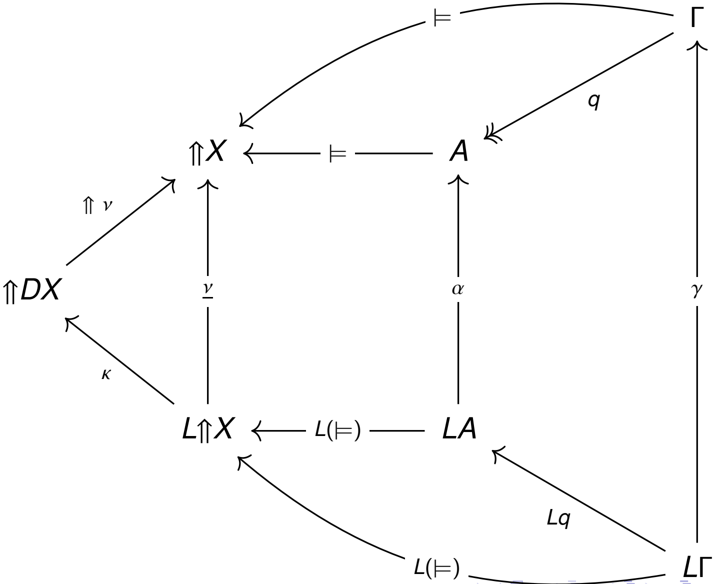
Fast learners

- **GPT-3, BERT:** simulate the sign

Meaning is a process



Signifier is a process



Lambek pregroups are Frobenius spiders

$$xx^{\ell} \leftarrow \iota \leftarrow x^{\ell}x \quad , \quad x^{r}x \leftarrow \iota \leftarrow xx^{r}$$

$$\Updownarrow$$

$$(xy \leftarrow uv) \;\vdash\; (xy \leftarrow xsv \leftarrow uv)\;,\;(xy \leftarrow uty \leftarrow uv)$$

Lambek pregroups are Frobenius spiders

$$xx^{\ell} \leftarrow \iota \leftarrow x^{\ell}x \quad , \quad x^{r}x \leftarrow \iota \leftarrow xx^{r}$$

$$\Updownarrow$$

$$(xy \leftarrow uv) \;\vdash\; (xy \leftarrow xsv \leftarrow uv) \;,\; (xy \leftarrow uty \leftarrow uv)$$

arXiv:2105.03038